MILLIMAN WHITE PAPER

Climate Change and Credit Risk

Managing credit transition risk in an uncertain climate

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In our paper, we introduce the main concepts of climate change and financial risks from climate change, followed by a brief introduction to approaches to modelling credit risk. We then investigate how credit risk models can be applied to climate change scenarios. The paper draws conclusions on this exercise and the next steps that insurers can take.

The paper is relevant for insurers with material exposures to credit risk, in particular providers of long-term business.

Introduction

The insurance industry has for a long time sought to identify the best practical methods for selecting investments to maximize returns within an acceptable level of credit risk. Since Markowitz's concept of diversification introduced in the 1950s, which states that, given a desired level of risk, an investor can optimize the expected returns of a portfolio through diversification, the selection of appropriate investments nowadays typically involves complex models to quantify credit risk for investment portfolios.

In more recent years, significant consideration has been given to the risks arising from climate change. Financial institutions and regulators are seen as key players which can help address risks from climate change, and the insurance industry through its investment portfolios and property and casualty business can play a major role.

Bringing these two areas together, credit risk has been identified as a key transmission channel of risks from climate change on firms' investment portfolios. In our paper, we consider how financial risks from climate change can impact insurers' exposure to credit risk, as measured through a known structural credit risk model—the Vašíček model. We consider how risks from climate change can be associated with variables and parameters in Vašíček's framework, and the credit risk of an asset portfolio and of individual assets.

What is climate change and why does it matter?

Greenhouse gas (GHG) emissions are changing our climate at a rapid pace. These changes come with some risks, like extreme weather events such as floods and heatwaves, and long-term term changes to natural systems such as sea temperatures and level. These risks are called 'physical risks.'

The response to climate change will bring changes to society and the economy. These changes also come with some risks, such as changes those governments, industries and consumers make in pursuit of a greener world, which in turn could prompt a reassessment of a wide range of asset values, a change in energy prices, and a fall in income and creditworthiness of some borrowers. These risks are called 'transition risks.'

CLIMATE-RELATED RISK DRIVERS

Financial institutions, such as banks, insurers and investment management firms, are exposed to climate change through the so-called transmission channels. They arise from the two types of climate risk drivers introduced earlier: physical risks and transition risks. Transmission channels can be macroeconomic and microeconomic.

PHYSICAL RISK DRIVERS

Physical risk drivers are changes in climate which impact economies. They can be categorised as chronic risks associated with gradual shifts in climate, or acute risks which are related to extreme weather events.

These drivers may appear with a significant time lag, and the frequency and severity of each type of risk may also vary considerably and become increasingly difficult to predict. Whilst human activity and decisions affect exposure to physical climate risks, the location, timing and magnitude of specific physical events cannot be controlled.

Acute physical risks generally consist of lethal heatwaves, floods, wildfires and storms, including hurricanes, cyclones and typhoons, as well as extreme precipitation.

Chronic physical risks generally include rising sea levels, rising average temperatures and ocean acidification. Extended periods of increased temperatures may lead to the further development of chronic climate events, such as desertification. Similarly, extended periods of increased average temperatures might impact the ecosystem, in particular agriculture.

TRANSITION RISK DRIVERS

Transition risk drivers are the societal changes arising from a transition to a low-carbon economy pursued to help mitigate the physical risks. They can arise through changes in public sector policies; innovation and changes in the affordability of existing technologies, e.g., which make renewable energies cheaper or allow for the removal of atmospheric GHG emissions; or investor and consumer sentiment towards a greener environment.

Transition risk drivers are global, although the specific nature of the risk driver will vary by economy. Examples of transition risk drivers are summarised below:

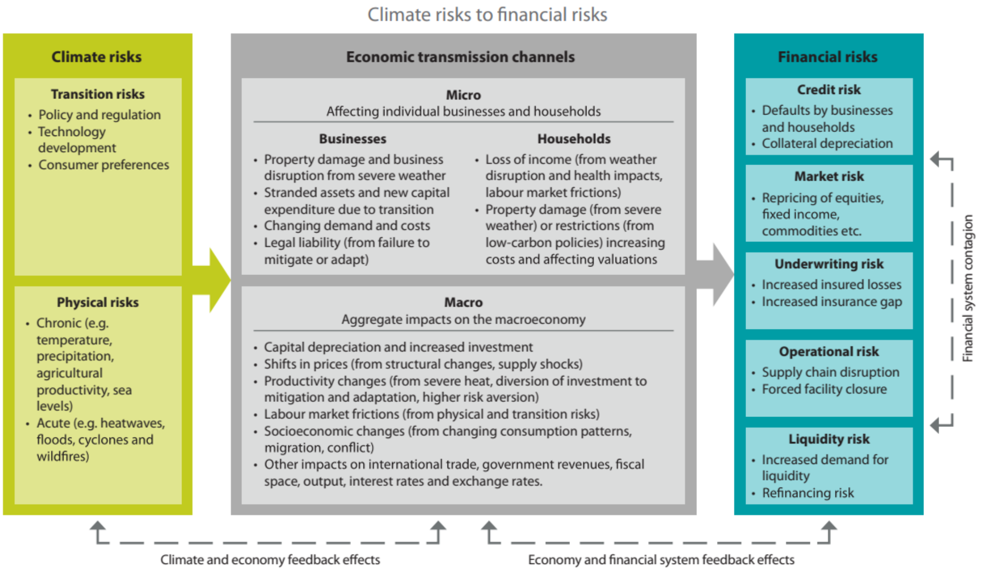
- Climate policies, such as the energy transition policies, pollution control regulation, policies on resource conservation and public subsidies as part of the Paris Agreement[1]
- Technology, such as energy-saving, low-carbon transportation and increasing use of non-fossil fuels
- Investor sentiment, such as increasing investor awareness and expectations with respect to climate change[2]
- Consumer sentiment, such as insurance customers requesting that their savings or investments be directed towards institutions with more climate-friendly policies

CLIMATE RISK TRANSMISSION CHANNELS AND ASSET PORTFOLIOS

So how do climate-related financial risks affect firms' asset portfolios, and in particular insurers' asset portfolios? As shown in **Figure 1**, risks from climate change impact all risks to which firms are exposed. This impact can occur directly through, for example, lower corporate profitability or the devaluation of assets, or indirectly, through macro-financial changes. At a risk category level, physical risks will impact physical property via depreciation of mortgage collaterals whilst transition risks will impact the business model of a debtor firm and ability to repay financing.

With invested assets of c.£1,400 billion in 2021[3], the life insurance sector in the UK will not be immune to climate-related financial risks. For example, the investments of providers of long-term annuity business include large portfolios of corporate bonds. Besides their exposure to longevity risk, these insurers are also exposed to market and credit risk due to their corporate bond investments.[4] These investments reveal exposures to the climate-related financial risks noted earlier via aggregate impacts on the macroeconomy (akin to systematic risks present in the wider economy) and businesses and households due to elevated levels of defaults and collateral depreciation (usually specific to individual assets or households; we can call these risks idiosyncratic risks).

FIGURE 1: CLIMATE RISK TRANSMISSION CHANNELS



Source: Network for Greening the Financial System, NGFS Scenarios for central banks and supervisors, September 2022

[1] United Nations. (2015), Paris Agreement. The signatory nations of the Paris Agreement committed to 'holding the increase in the global average temperature to well below 2°C above pre-industrial levels and pursuing efforts to limit the temperature increase to 1.5°C above pre-industrial levels.' Retrieved 16 February 2023 from https://unfccc.int/sites/default/files/english_paris_agreement.pdf.

[2] A growing number of investors are incorporating climate risk considerations into their investment decisions, potentially reflecting growing pressure from non-governmental associations and environmental groups. The risk profile and valuation of debt and equity investments of corporates exposed to climate change will be impacted as investors undertake a reassessment of their investment decisions.

[3] Prudential Regulation Authority, Bank of England. (2021, May 24). Results of the 2021 Climate Biennial Exploratory Scenario (CBES). Chart 4.8 Invested assets by insurer type. Retrieved 16 February 2023 from https://www.bankofengland.co.uk/stress-testing/2022/results-of-the-2021-climate-biennial-exploratory-scenario.

[4] For the avoidance of doubt, a distinction is made in our paper between credit risk like counterparty default risk, and credit spread risk, due to changes in the market values of corporate bonds. Our focus is the former.

CHARACTERISTICS OF CLIMATE-RELATED FINANCIAL RISKS

Climate-related risks have distinctive characteristics which warrant consideration.[5] These characteristics include the far-reaching impact in breadth and magnitude, an uncertain and long-term time horizon, and the dependency on short-term action.

The Prudential Regulation Authority (PRA) also highlighted these characteristics in a recent publication: climate-related financial risks are systemic, simultaneously uncertain and yet totally foreseeable, and the size and balance of the future risks we have will be determined by actions we start to take now.[6]

Together, these factors give rise to a material level of uncertainty as to how climate risk drivers and their impacts will evolve. This uncertainty is driven by assumptions around future emissions pathways and the impact which these have on physical hazards, interactions between natural systems, future paths of policy, technological advances, and consumer and market sentiment.

CHALLENGES WITH ASSESSING FINANCIAL RISKS FROM CLIMATE CHANGE

There is a vast literature discussing the challenges in assessing financial risks from climate change. Two often mentioned challenges include the lack or limited availability of empirical data and the long-time horizons required for modelling climate-change impacts.

In two relatively recent research papers discussing credit risk from climate change[7], the United Nations Environment Finance Initiative highlights the following challenges with assessing climate-related transition risks and physical risks for financial institutions (noting that the papers focus on banks' loan portfolios):

- Long-time horizons for transition impacts. Long-term modelling horizons would likely be required, which can be challenging for financial institutions.
- Limited empirical data exists to measure the strength of the climate-credit risk relationship. A possible way to tackle this challenge is to make the best use of informative insights from climate scenarios on the potential economic effects under different scenarios.
- Risks from climate change vary across sectors and industries, and methodologies will need to be flexible to accommodate different risks.
- Data analysis and granularity or lack of data for assessing physical risks for some locations and climate parameters.

In this context, interested stakeholders including regulators, industry bodies and non-government organisations are stepping up their efforts in tackling some of these challenges. In the UK, the PRA published Supervisory Statement 3/19 on Enhancing banks' and insurers' approaches to managing the financial risks from climate change, and has included financial risks from climate change as a priority in its Dear CEO letter.[8]

More recently, regulators focused on scenario-based approaches to assessing financial risks from climate change; the Bank of England ran the 2021 Climate Biennial Exploratory Scenario: Financial risks from climate change (2021 CBES), which explored the resilience of the UK financial system to the physical and transition risks associated with different climate pathways. It is worth noting that the 2021 CBES is not just a one-off, but something which will inform future regulatory expectations.

Although scenario-based approaches still require climate-related data—noting that they don't create data or reduce uncertainty—they explore outcomes consistent with available data. Adopting the same scenarios across markets, like the 2021 CBES, facilitates further analyses and insights into impacts of climate change.

Introduction to credit risk modelling

Through their large corporate bond holdings, providers of long-term annuity business have exposure to credit risks which may be influenced by physical and transition risks via transmission channels, as shown in **Figure 1**. Exposures to physical and transition risks are caused by climate-related impacts on the macroeconomy and individual assets and households.

A key metric in gauging the credit risk in the context of corporate bonds is their credit rating and evolution through time (or projection periods) of rating changes and defaults. Changes to credit ratings can be translated into changes in market values of corporate bonds, although in our paper we focus on the former aspect of credit risk—climate-change impacts on credit ratings.

[5] Network for Greening the Financial System. (2019, April). A call for action: Climate change as a source of financial risk. Retrieved 16 February 2023 from https://www.ngfs.net/sites/default/files/medias/documents/synthese_ngfs-2019_-_17042019_0.pdf.

[6] Prudential Regulation Authority, Bank of England. (2021, October 28). Climate-related financial risk management and the role of capital requirements. Retrieved 16 February 2023 from https://www.bankofengland.co.uk/-/media/boe/files/prudential-regulation/publication/2021/october/climate-change-adaptation-report-2021.pdf.

[7] Colas, J., Khaykin, I., Pyanet, A., & Westheim, J. (2018). Extending our horizons: Assessing credit risk and opportunity in a changing climate, PART 1: Transition-related risks & opportunities. United Nations Environment Finance Initiative. Retrieved 16 February 2023 from https://www.unepfi.org/wordpress/wp-content/uploads/2018/04/EXTENDING-OUR-HORIZONS.pdf; Connell, R., Firth, J., Baglee, A., Haworth, A., Steeves, J., Fouvet, C., & Hamaker-Taylor, R. (2018). Navigating a new climate: Assessing credit risk and opportunity in a changing climate, PART 2: Physical risks and opportunities. United Nations Environment Finance Initiative. Retrieved 17 February 2023 from https://www.unepfi.org/wordpress/wp-content/uploads/2018/07/NAVIGATING-A-NEW-CLIMATE.pdf.

[8] Gerken, C. & Khan, S. (2023, January 10). Insurance supervision: 2023 priorities. Prudential Regulation Authority, Bank of England. Retrieved 17 February 2023 from https://www.bankofengland.co.uk/-/media/boe/files/prudential-regulation/letter/2023/insurance-supervision-2023-priorities.pdf?la=en&hash=9ABF6B8EB633A02308D0D9692374867A3109E8ED.

In this section, we summarise two of the most known credit risk models—structural and reduced-form. We then cover in more detail a type of structural model and consider its application to credit ratings in the context of climate changes.

We note that there is a vast literature describing credit risk models, and without going into too much detail we reference a more recent paper authored by the Centre for Central Banking Studies at the Bank of England providing a summary of credit risk models.[9]

STRUCTURAL MODELS

A model of default is known as a structural, or firm-value model, when it attempts to describe the mechanism by which default takes place. These models use the evolution of a firm's 'structural' variables, such as asset and debt values, to determine probabilities of (economic) conditions under which borrowers are expected to default. Default occurs whenever a stochastic variable representing firms' asset values falls below a threshold representing liabilities. The Merton model (1974), one of the first,[10] assumes a firm's asset value evolves over time, e.g., through a simple diffusion process. Starting from the geometric Brownian motion, the following equation summarises the Merton model:

EQUATION 1.

$$dA(t) = \mu A(t)dt + \sigma A(t)dW(t) \xRightarrow{It\hat{o}'s\ lemma\ and\ t=T} lnA(T) = lnA + \mu T - \frac{1}{2}\sigma^2 T + \sigma\sqrt{T}X,$$

where:

- T is maturity
- $W(t)$ is the value of a standard Brownian motion at time t
- Asset values at time t (denoted A(t)) are log-normal distributed
- μ and σ^2 are the instantaneous expected rate and instantaneous variance of asset return, respectively
- X represents the return on a firm's asset, given by $X = \frac{W(T)-W(0)}{\sqrt{T}}$

Where a firm's liabilities (noted with L(t)) are known, default probabilities can be determined across the Standard Normal distribution. Using these assumptions, under the Merton model, a firm's probability of default (defined as occurring when the value of the assets is less than liabilities) can be determined as follows:

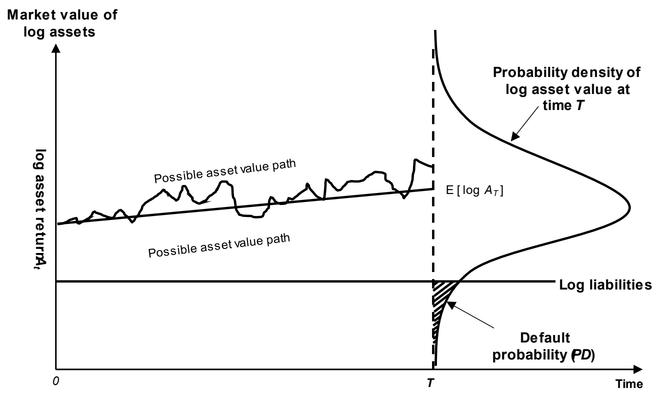
EQUATION 2.

$$P(A(T) < L(T)) = \Phi\left(\frac{L(T) - E(A(T))}{\sigma(A(T))}\right) = \Phi\left(\frac{\ln(L(T)) - \ln(A(t) - \left(\mu - \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}\right) = \Phi\left(-\frac{\ln(A(t) + \left(\mu - \frac{\sigma^2}{2}\right)(T-t) - \ln(L(T))}{\sigma\sqrt{T-t}}\right)^{11},$$

where $\Phi(x)$ is the cumulative density function of the Standard Normal distribution.

The Merton model can be graphically illustrated as shown in **Figure 2**:

FIGURE 2: ILLUSTRATION OF THE MERTON MODEL



The probability of default can be quantified if we know the parameters in **Equation 2**, i.e., a firm's assets (and liabilities) or their distribution(s) (along parameters μ and σ^2). An alternative, simple and pragmatic approach is to consider Z-scores across the Standard Normal distribution and derive the corresponding probability of default, as in **Equation 2**.

The Merton model can be extended to include credit rating changes. This involves stating that there are credit rating upgrade/downgrade thresholds in addition to the default threshold. A firm's asset value relative to these thresholds determines its future credit rating, as illustrated in **Figure 3**.[12] A practical application of this approach is for transition matrices,[13] discussed later in our paper.

[9] Chatterjee, S. (2015).Modelling Credit Risk. Centre for Central Banking Studies, Bank of England. Retrieved 17 February 2023 from https://www.bankofengland.co.uk/-/media/boe/files/ccbs/resources/modelling-credit-risk.pdf.

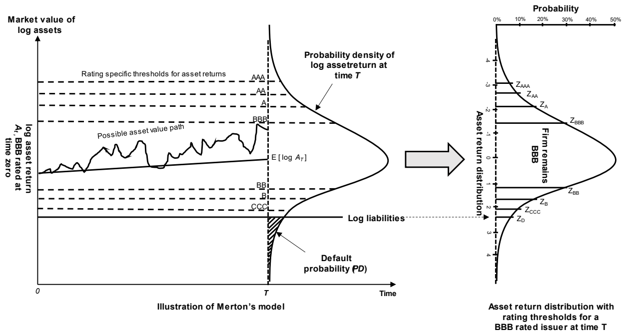
[10] Merton, R.C. (1974). On the pricing of corporate debt: The risk structure of interest rates. The Journal of Finance, 29(2), 449-470.

[11] The term $\frac{\ln(A(t) + \left(\mu - \frac{\sigma^2}{2}\right)(T-t) - \ln(L(T))}{\sigma\sqrt{T-t}}$ is also known as "distance-to-default" (DD) or "Z-score". It represents the number of standard deviations that A is away from default on a Standard Normal distribution.

[12] Examples of sources of Merton's model representation on a Standard Normal distribution are shown in Figure 7 Probability of default, Centre for Central Banking Studies (Bank of England), Modelling Credit Risk, 2015, and Chart 3.3 Model of firm value and generalized credit quality thresholds, RiskMetrics Group, CreditMetrics Technical Document, 2007.

[13] For example, RiskMetrics Group, CreditMetrics™ Technical Document, 2007 (first published in 1997).

FIGURE 3: ILLUSTRATION OF THE MERTON MODEL WITH CREDIT RATING THRESHOLDS



So far, in the Merton model, we considered credit risk for an individual asset (also known as the 'bottom-up' approach). Large institutional investors, such as life insurers, would typically require a portfolio-level approach (also known as the 'top-down' approach). Structural models such as the Vašíček model are well-suited for this.

How can we extend the Merton model to a portfolio? Assuming one knows a firm's probability of default, how can we estimate a portfolio probability of default? This is the question which Oldřich Vašíček tried to answer, by adapting Merton's single asset model to a portfolio of loans.[14] For a firm denoted *i* (with *i* = 1, …, n), **Equation 1** can be re-written as:

EQUATION 3.

$$lnA_i(T) = lnA_i + \mu_i T - \frac{1}{2}\sigma_i{}^2 T + \sigma_i\sqrt{T}X_i$$

Several assumptions are required under Vašíček's approach:

- All asset log returns are described by a Wiener process. In other words, all asset values are log-normal distributed, like Merton's approach.
- All assets have the same probability of default p.
- All assets are of equal amounts.
- Any two of the assets are correlated with a coefficient ρ (i.e., assets are equi-correlated).[15]

The assumption that all assets have the same probability of default implies similar credit risk profile. In the context of transition matrices, assets in a row have the same probability of default; in other words, assets with the same initial credit rating have the same probability of default.

In statistical theory, variables X_i belong to the equi-correlated Standard Normal distribution. Variables X_i can thus be represented as a linear combination of two other jointly Standard Normal variables random variables Z and Y_i such that:

EQUATION 4.

$$X_i = Z\sqrt{\rho} + Y_i\sqrt{1-\rho}, \text{ i = 1,…, n}^{16}$$

where Z, Y_1, Y_2, …, Y_n are mutually independent Standard Normal variables, and n is the number of firms in a portfolio.

In statistical theory, ρ is used instead of $\sqrt{\rho}$, which is the notation in Vašíček's framework. For this to be true, we need to demonstrate that ρ in Equation 4 can only take positive values. Although Vašíček's paper missed this proof, it is relatively straightforward. For convenience, we included it in Appendix C.

With each firm's asset return X_i of the form $X_i = Z\sqrt{\rho} + Y_i\sqrt{1-\rho}$, variable Z is common across the entire portfolio of assets, whilst variables Y_i are the i[th] firm's specific variables j <> i. Vašíček denotes variable Z as a portfolio common factor (for instance, some measure of the state of the economy, or macroeconomic variable) and variable Y_i as each firm's specific risk.

Variable Z can be seen as a measure of the 'credit cycle;' in good years it will be positive, implying a lower-than-average default rate. In bad years, the reverse will be true. A Z value of 0 is equivalent to the (long-term) average year.

The asset correlation ρ is an important driver of credit risk (it is a measure of the likelihood of the joint default). A portfolio with high correlations produces greater default oscillations over an economic cycle, compared with a portfolio with lower correlations. In good years, a portfolio with high correlations will produce fewer defaults than a portfolio with low correlations, whilst in bad years, the opposite is true, and high correlations are creating more defaults.

[14] Vašíček , O.A. (1987). Probability of loss on loan portfolio. Retrieved 17 February 2023 from https://www.moodysanalytics.com/-/media/whitepaper/before-2011/02-12-87-probability-of-loss-on-loan-portfolio.pdf.

[15] A positive asset correlation will result in a higher joint probability of default.

[16] This result follows from the statistical properties of jointly equi-correlated Standard Normal variables, which stipulates that any two variable X_i and X_j are bivariate Standard Normal with corelation coefficient ρ if there are two independent Standard Normal variables Z and Y for which $X_i = Z$ and $X_j = \rho Z + \sqrt{1-\rho^2}Y$, with ρ a real number in [-1, 1]. See example 5.36 at https://www.probabilitycourse.com/chapter5/5_3_2_bivariate_normal_dist.php.

The final step in Vašíček's construct is the derivation of a firm's probability of default, conditional on the common factor Z (or in other words, conditional on the state of the economy):

EQUATION 5.

$$P(firm\ i\ defaults \mid Z) = \Phi\left(\frac{x_i - Z\sqrt{\rho}}{\sqrt{1-\rho}}\right).$$

Where:

- Φ(•) represents the standard normal cumulative distribution function.
- x_i is i[th] firm's probability of default, unconditional on systematic factor Z (for example, probabilities of default published by credit rating agencies).

REDUCED-FORM MODELS

Reduced-form models (also known as survival models) typically assume an exogenous cause of default. They model default as a random event without any focus on a firm's balance sheet. This random event of default can be described as a Poisson event. As Poisson models look at the arrival rate, or intensity, of a specific event, this approach to credit risk modelling is also referred to as default intensity modelling.

Mathematically, the default intensity λ_t can be modelled as a function of a vector of explanatory covariates X_t. That is, if β is a vector of parameters defining the correlation with the explanatory factors, then a reduced-form model has the basic form:

EQUATION 6.

$$\lambda_t = \lambda(X_t) = \lambda_{0,t} e^{\beta \times X_t},$$

where $\lambda_{0,t}$ is the baseline function.

Assuming this intensity of default is constant over the time interval (t, t+1), the probability of default (PD_t) in this interval is given by:

EQUATION 7.

$$PD_t = 1 - e^{-\lambda(X_t)}.$$

Note that in an intensity model, the default correlation between obligors in the portfolio comes only from the common economic factors.

An example of a reduced-form model is the Jarrow-Lando-Turnbull (JLT) model. The model captures the joint dynamics of default and credit rating transitions, by reference to a deterministic transition matrix, adjusted via a risk premium process modelled by a Cox-Ingersoll-Ross dynamic.

Application of the Vašíček model to transition matrices

A transition matrix is a two-dimensional representation of a corporate bond portfolio over a period (usually a year). At the start of the period, assets are grouped according to their credit rating. Historical transition matrices, such as those published by credit rating agencies give unconditional historical transition rates.

The quality of asset returns in Merton's and Vašíček's frameworks are reflected in their credit rating at the beginning of the period, as illustrated **Figure 3** above. Under the Merton framework, when a firm's liabilities are known, with asset returns following the Standard Normal distribution, probabilities of default can be determined across the distribution. This framework can be extended to include rating changes over the period—in addition to the default threshold, there are credit rating upgrade/downgrade thresholds as well. The firm's asset value relative to these thresholds determines its future credit rating at the end of the period.

We can extend the application of the Merton framework to the Vašíček framework. Joint probabilities (or transition rates) for a large portfolio of corporate bonds can be derived with the knowledge of the equi-correlation between bonds. We can derive transition rates in a transition matrix based on probabilities of default, conditional on the common factor Z derived in **Equation 8** below, where rating thresholds (or 'bins') are calibrated based on published transition matrices, and with an appropriately selected parameter ρ.

An example of the approach can be found in Belkin and Suchower,[17] which applies the Vašíček framework to transition matrices. Fitted transition rates for the G-to-g credit rating (where G is the credit rating at the beginning of the year, and g the credit rating at the end of the year) can be calculated using the following formulae:

EQUATION 8.

$$Transition\ rate\ (for\ cell\ G, g+1) =$$

$$\Phi\left(\frac{x_{g+1}^G - Z\sqrt{\rho}}{\sqrt{1-\rho}}\right) - \Phi\left(\frac{x_g^G - Z\sqrt{\rho}}{\sqrt{1-\rho}}\right).$$

Where:

- Φ(•) represents the standard normal cumulative distribution function, with Φ(∞) = 1 and Φ(-∞) = 0
- G is the credit rating at the beginning of the year (G = AAA, AA, …, CCC/C)
- g is the credit rating at the end of the year (g = AA, A, …, CCC/C, and g+1 = AAA, AA, …, B).

[17] Belkin, B. & Suchower, S. (1998). A one-parameter representation of credit risk and transition matrices. CreditMetrics Monitor, third quarter, p. 46. Retrieved 17 February 2023 from https://www.z-riskengine.com/media/hqtnwlmb/a-one-parameter-representation-of-credit-risk-and-transition-matrices.pdf.

To derive fitted transition rates for a portfolio of corporate bonds, the method in Belkin and Suchower requires the following steps:

- **Step 1.** Select an appropriate long-term average annual transition matrix. This is required to derive the Standard Normal bins for each cell in a transition matrix. The long-term average annual matrix will correspond to a Z value of zero.
- **Step 2.** Apply any required adjustments to the transition matrix, for example, adjustments for non-rated transitions.
- **Step 3.** Derive Standard Normal bins, including any adjustments needed. For example, for zero transition rates, non-zero values are required to derive meaningful values for the bins.
- **Step 4.** Select an appropriate value for the asset correlation variable. The statistical theory underlying the Vašíček model requires a single asset correlation for an entire asset portfolio represented in a transition matrix. This is a useful simplification; in practice firms may split their portfolios and use several asset correlations, by industry, sector and/or geography.
- **Step 5.** Select (or calculate) Z variables for the projection period.
- **Step 6.** Derive projected annual transition matrices based on the Standard Normal bins derived in Step 3, the asset correlation derived in **Step 4** and variables Z derived in **Step 5**.

In addition to projected annual transition matrices, we can also derive credit ratings pathways for corporate bonds (or firms) with the same credit rating at the start of projection. Recall that a bond's (or firm's) credit riskiness is given by variable X_i, i = 1, …n.

As shown in **Equation 4**, we require projections of systematic variable Z, idiosyncratic variables Y_i and an asset correlation assumption to derive X_i. The next step is to project transition probabilities as shown in **Equation 9** below:

EQUATION 9.

$$P(transition\ rate\ firm\ i) = \Phi(x_i) =$$

$$\Phi\left(z\sqrt{\rho} + y_i\sqrt{1-\rho}\right), \text{ i =1,…,n}$$

and mapping transition probabilities to the corresponding credit rating in projected transition matrices.

In the next section, we take a closer look at the Bank of England's 2021 climate scenario stress test. The historical data and scenario projections included in the Bank of England's stress test can be applied to the Vašíček's framework.

THE 2021 CBES SCENARIOS

In 2021, the Bank of England carried out the first stress testing exercise to test the UK largest banks' and insurers' resilience to climate-related risks, 2021 CBES.[18] The 2021 CBES scenario specification builds upon a subset of the Network for Greening the Financial System climate scenarios and were designed to explore the resilience of the UK financial system to the physical and transition risks associated with different climate pathways.

The 2021 CBES considered two routes to net zero GHG emissions, which primarily explore transition risks from climate change:

- Early Action scenario: the transition to a net zero emissions economy starts in 2021. The overall impact on GDP growth is muted, particularly in the latter half of the scenario.
- Late Action scenario: the implementation of policy to drive transition is delayed until 2031 and is then more sudden and substantial.

A third scenario was also included:

- No Additional Action scenario: this scenario explores physical risks from climate change, assuming there are no new climate policies introduced beyond those already implemented.[19]

The 2021 CBES also includes a baseline scenario (or 'counterfactual'), which is based on two data points: 2040 and 2050. For the first 10 years of the projection (2021-2030), the baseline scenario is like the Late Action scenario.

Projected variables included in the 2021 CBES are split in four categories for each of three scenarios as follows:

1. Macro variables: gross domestic product (GDP), consumer price index (CPI), unemployment rates, bank rates, residential property prices, etc.
2. Financial variables: equity prices, bond yields, interest rates, bond spreads, etc.
3. Transition variables: carbon prices, energy demand, energy prices, cost of coal, car prices, etc.
4. Physical variables: temperatures, wind speeds in various areas, precipitations, soil moisture, etc.

The scenario projections period in the 2021 CBES is 30 years, from 2021 to 2050.[20] For most of the variables, historical data

[18] Prudential Regulation Authority, Bank of England. (2021, June 8). Key elements of the 2021 Climate Biennial Exploratory Scenario: Financial risks from climate change. Retrieved 17 February 2023 from https://www.bankofengland.co.uk/stress-testing/2021/key-elements-2021-biennial-exploratory-scenario-financial-risks-climate-change.

[19] As indicated in the 2021 CBES, climate scientists' projections suggest that absent a rapid transition, some physical risks will crystallise in the period to 2050, but the most material shocks would occur later in the century. To ensure the "No Additional Action" scenario captures these more severe risks, the calibration is based on the level of physical risk that could be prevalent between 2050 and 2080 in the absence of further policy action.

[20] The No Additional Action scenario captures material risks which are expected to occur beyond 2050.

covers the period 2000-2020, although for most physical variables there is only one historical data point included (for 2020).

The variables included in the scenario projections present several characteristics which mean we can consider them in the Vašíček model (as systematic variables Z or idiosyncratic variables Y_i):

- The 2021 CBES data and scenario projections are publicly available.[21]
- All participating firms (banks, insurers) used the same dataset.
- Several variables are global (for example, purchasing power parity-weighted world real GDP, or 'PPP global GDP'), whilst others are industry-specific (for example, Crop and animal production), specific to the financial sector (for example, corporate bond spreads), or specific to physical risks (for example, sea levels and temperatures).

The data for the systematic and idiosyncratic variables in Vašíček's framework need to satisfy certain criteria:

- Data for the systematic variables is global (like the data in the long-term average transition matrix).
- Data for the idiosyncratic variables is asset-specific, for example, industry-specific data.
- The two data sets for systematic and idiosyncratic variables need to be independent.
- The data for the variables is dimensionless. For example, taking annual changes instead of absolute amounts or units.

From the variables list included in the 2021 CBES, we select PPP global GDP for systematic variables and Crop and animal production for idiosyncratic variables. The absolute values and annual changes[22] for all scenarios in the 2021 CBES (Baseline, Early Action, Late Action and No Additional Action) are shown in **Figure 4** and **Figure 5**, respectively:

FIGURE 4. PPP GLOBAL GDP, 2021 CBES SCENARIOS

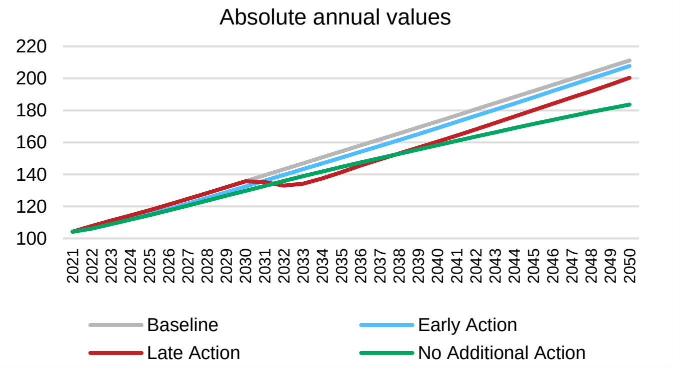


FIGURE 4. PPP GLOBAL GDP, 2021 CBES SCENARIOS (CONTINUED)

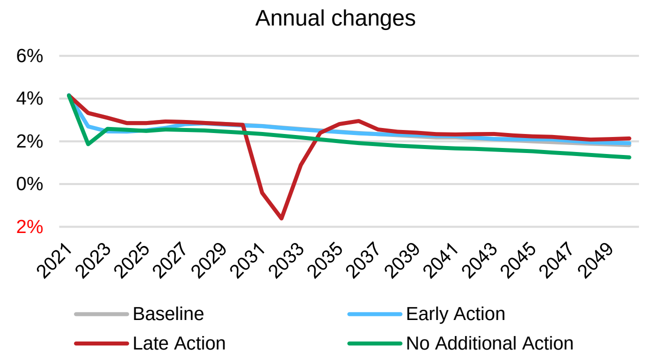
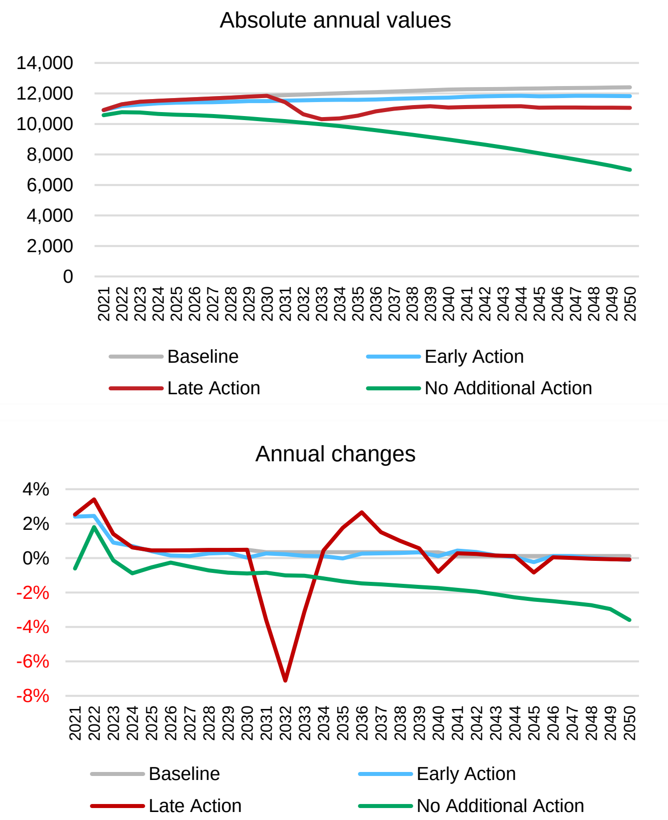


FIGURE 5. CROP AND ANIMAL PRODUCTION, 2021 CBES SCENARIOS



Note: In the 2021 CBES, the baseline scenario is based on two data points (2040 and 2050) and set equal to the Late Action scenario for the first 10 years of the projection (2021-2030). For all other year in the projection, we applied linear interpolation.

[21] Prudential Regulation Authority, Bank of England. (2021, June 8). Key elements of the 2021 Climate Biennial Exploratory Scenario: Financial risks from climate change. Retrieved 17 February 2023 from https://www.bankofengland.co.uk/-/media/boe/files/stress-testing/2021/variable-paths.

[22] In Merton's formula, asset returns are assumed to be log-normally distributed. By extension, in the Vašíček's framework we require that variables used to derive or calibrate credit risk variables X, Z and Y_i are also log-normally distributed. As a result, annual changes of variables in the 2021 CBES are calculate as natural logarithm of the ratios of consecutive absolute variables, e.g., $LN\left(\frac{GDP(year\ t+1)}{GDP(year\ t)}\right)$.

TRANSFORMING THE 2021 CBES DATA

The Z and Y_i variables in the Vašíček framework are Standard Normal. It follows that the annual changes in PPP global GDP and Crop and animal production need to be standardized.

To standardize a variable, its deviation from the population mean is divided by the population standard deviation. The process involves fixing the historical period at inception and rolling it forward over time. For example, the value of a variable in 2021 is standardized based on its historical mean and standard deviation between 2001 and 2020; its value in 2022 is based on historical data between 2002 and 2021 and so on.

The standardization processes for PPP global GDP and Crop and animal production variables are different, and we discuss each in the next two sections.

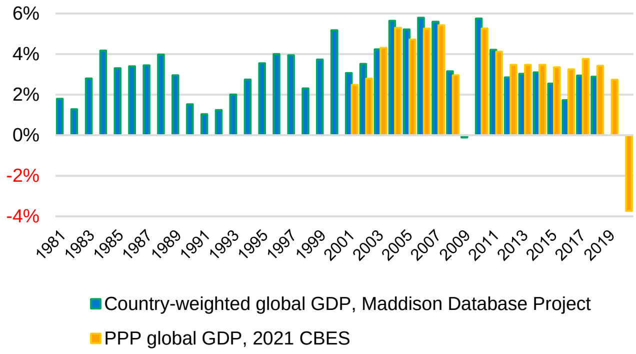
PPP global GDP variables

For the PPP global GDP variables, we apply the method described in Belkin and Suchower to derive fitted transition rates for a corporate bond portfolio. The method requires us to select an appropriate historical long-term average annual transition matrix, which corresponds to a Z value of zero. A suitable choice is the global corporate average annual matrix 1981-2020 from S&P.[23]

The historical period for the long-term average transition matrix needs to be consistent with the historical period of Z variables. The historical annual changes for PPP global GDP in the 2021 CBES scenarios are over the period 2001-2020, whilst the historical data in the long-term transition matrix is over the period 1981-2020. Therefore we require additional 20 years of historical global GDP. These can be sourced from the Maddison Project Database,[24] a provider of quantitative information on global economic growth and income levels. A suitable variable in the database is the country-weighted real global GDP.

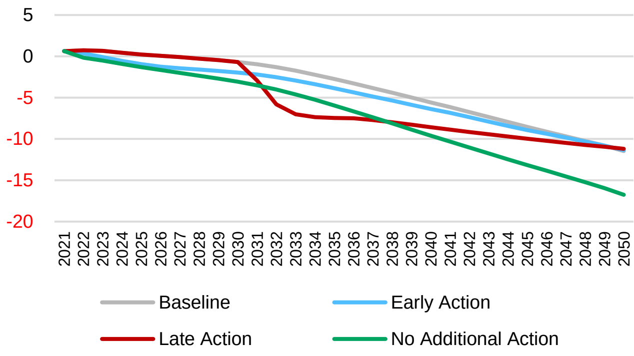
The two historical data sets—PPP global GDP and country-weighted global GDP—are shown in **Figure 6** below. We note that for the overlapping period 2001-2018, the two data sets are relatively close.

FIGURE 6. ANNUAL CHANGES IN GLOBAL GDP, 2021 CBES AND MADDISON PROJECT DATABASE



We can now derive annual Z variables for the projection period, shown cumulative in **Figure 7** below. By way of illustration, the Z value in 2021 in the Early Action scenario is 0.64, which is equal to annual return in 2021 of 4.16% less historical average of 3.10%, divided by the historical standard deviation of 1.65% for the period 1981 to 2020.

FIGURE 7. CUMULATIVE Z VARIABLES, ANNUAL CHANGES IN PPP GLOBAL GDP, 2021 CBES SCENARIOS



Systematic risk in the Baseline scenario increases over time; in other words, PPP global GDP increases by less than historical average in all future years.

In the context of Vašíček's framework, Z variables are a measure of invested asset losses. We can compare the cumulative Z variable pathways with projected investment losses on insurers' investment assets in the results of the 2021 CBES exercise,[25] with results being relatively close. The full comparison is included in **Appendix A**.

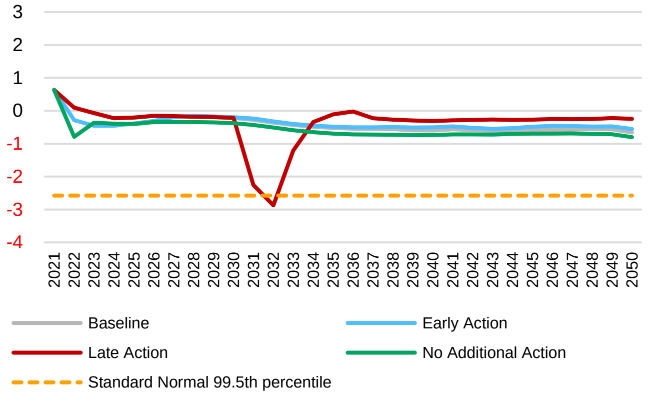
[23] S&P Global Ratings. (2021). Default, transition, and recovery: 2020 annual global corporate default and rating transition study. Table 21 'Global average default rates (1981-2020) (%).' Retrieved 17 February 2023 from https://www.spglobal.com/ratings/en/research/articles/210407-default-transition-and-recovery-2020-annual-global-corporate-default-and-rating-transition-study-11900573.

[24] The Maddison Project Database. (2020). Retrieved 17 February 2023 from https://www.rug.nl/ggdc/historicaldevelopment/maddison/releases/maddison-project-database-2020?lang=en.

[25] Prudential Regulation Authority, Bank of England. (2021, May 24). Results of the 2021 Climate Biennial Exploratory Scenario (CBES). Retrieved 16 February 2023 from https://www.bankofengland.co.uk/stress-testing/2022/results-of-the-2021-climate-biennial-exploratory-scenario.

The results also show a noticeably large increase in systematic risk in 2031 and 2032 under the Late Action scenario. Since annual Z variables are Standard Normal, we can compare these to percentiles across the Standard Normal distribution, for example the 99.5th percentile (equivalent to a 'one in 200 years' event). As shown in **Figure 8** below, the increase in systematic risk in 2031 and 2032 is as severe as the 99.5th percentile.

FIGURE 8. ANNUAL Z VARIABLES, CHANGES IN PPP GLOBAL GDP (2021 CBES SCENARIOS)



As a final observation, it could be argued that the severity of the three scenarios is dependent on the Baseline scenario. As noted above, the Baseline scenario is also adverse. Arguably, one could consider results where Z variables in the Baseline scenario are set to zero, with Z values in all other scenarios based on 'translated' annual changes, i.e., deduct annual changes in the Baseline scenario from each of the other three scenarios. We will consider this approach in the next section.

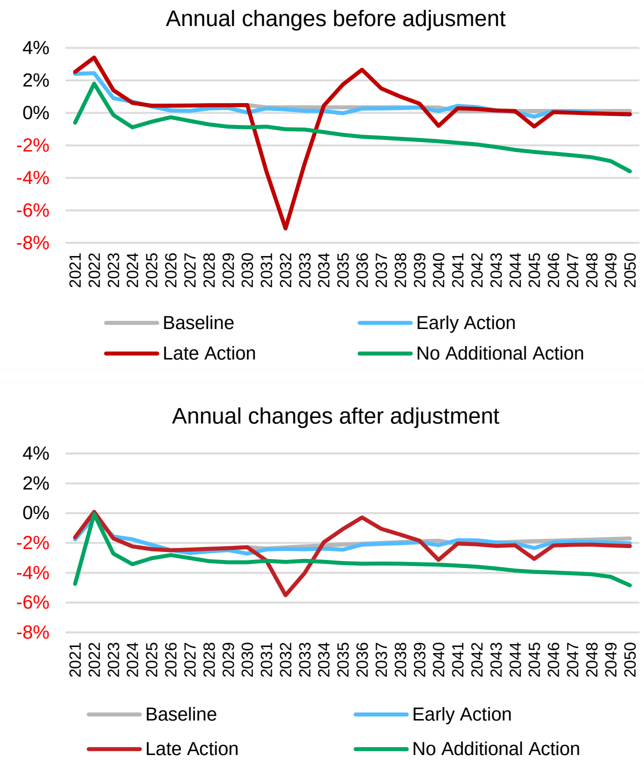
Crop and animal production variables

For Crop and animal production variables, we can project idiosyncratic variables Y_i based on historical values provided in the 2021 CBES exercise.

We note that under the Vašíček framework, idiosyncratic variables Y_i and systematic variables Z need to be independent. A simple check for independence is looking at the correlation of historical annual changes in PPP global GDP and Crop and animal production. The correlation is 27%, therefore we adjust the annual changes in Crop and animal production by deducting annual changes in PPP global GDP.[26] The updated correlation is now 2%.

The annual changes in Crop and animal production before and after the adjustment are shown in **Figure 9**.

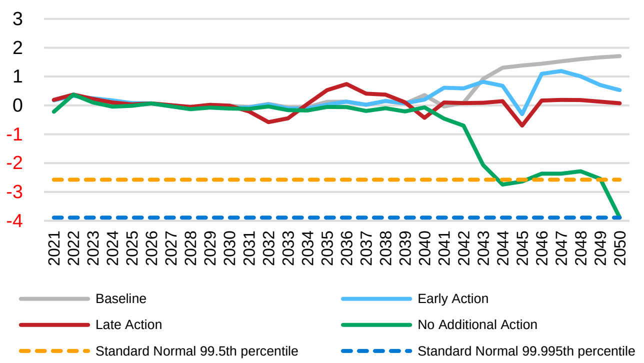
FIGURE 9. PROJECTED ANNUAL CHANGES IN CROP AND ANIMAL PRODUCTION, 2021 CBES SCENARIOS



In the remainder of our paper, references to Crop and animal production variables mean the variables are after the adjustment for annual changes in PPP global GDP.

The results show a continuing increase in idiosyncratic risk in the No Additional Action scenario. We can also compare annual Y_i variables to percentiles across the Standard Normal distribution. As shown in **Figure 10** below, the increase in idiosyncratic risk beyond 2043 is as severe as the 99.5th percentile, and as severe as the 99.995th percentile (equivalent to a 'one in 20,000 years' event) by 2050.

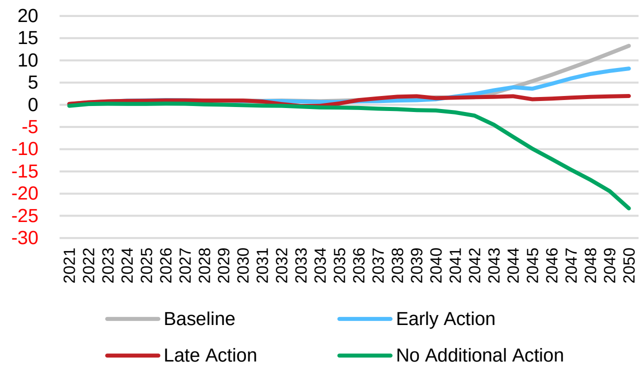
FIGURE 10. ANNUAL Y VARIABLES FOR STANDARDISED ANNUAL CHANGES OF CROP AND ANIMAL PRODUCTION, 2021 CBES SCENARIOS



[26] To remove correlations between two sets of data, other approaches are possible, such as Principal Component Analysis.

The resulting cumulative Y_i variables for Crop and animal production are shown in **Figure 11** below:

FIGURE 11. CUMULATIVE Y_i VARIABLES, ANNUAL CHANGES IN CROP AND ANIMAL PRODUCTION, 2021 CBES SCENARIOS



The results show that idiosyncratic risk reduces in all but the No Additional Action scenarios. By the end of the projection period, it increases sharply in the No Additional Action scenario. This is not unexpected: the 2021 CBES results do mention Crop and animal production being one of the three sectors most exposed to physical risks (Manufacturing, Crop and animal production, and Forestry and fishing), and physical risks expected to manifest mostly in the No Additional Action scenario.

Modelling results

Having derived the Z and Y_i variables for all scenarios, we can now project portfolio-level transition matrices and credit ratings pathways, for which we introduced a six steps approach. The results for the first three steps are shown in **Figure 12** below.

For step 4, we require an appropriate asset correlation assumption. There are several alternative approaches to derive an appropriate assumption, which include:

1. Derive an implicit asset correlation from historical transition matrices data, based on distributions of variables Z and Y_i. This approach requires access to full historical annual transition matrices.
2. Set the assumption based on expert judgement (for example, based on correlations between equity indices); however, this approach is rather complex and difficult to implement in our modelling).
3. Select the assumption based on public information. There are not many public sources available; however, we note two of them:
   - A paper[27] by the Bank for International Settlements which discusses the application of the Vašíček model to counterparty default for banks. A range of 12% to 24% for the asset correlation is given in the paper.
   - A CreditMetrics technical document,[28] which gives a range of 20% to 35% for the asset correlation.

Based on the above, an asset correlation of 25% seems reasonable.

The projected Z and Y_i variables required in step 5 were derived in the previous section.

Finally in step 6, we can project portfolio-level transition matrices and credit rating pathways for each scenario.

FIGURE 12. LONG-TERM AVERAGE TRANSITION MATRIX AND ASSOCIATED STANDARD NORMAL BINS

	From/to	AAA	AA	A	BBB	BB	B	CCC/C	D	NR (not rated)
Step 1. Global Corporate Average Transition Rates (1981-2020)	**AAA**	87.10%	9.10%	0.50%	0.10%	0.10%	0.00%	0.10%	0.00%	3.10%
	AA	0.50%	87.20%	7.80%	0.50%	0.10%	0.10%	0.00%	0.00%	3.90%
	A	0.00%	1.60%	88.60%	5.00%	0.30%	0.10%	0.00%	0.10%	4.40%
	BBB	0.00%	0.10%	3.30%	86.50%	3.60%	0.40%	0.10%	0.20%	5.90%
	BB	0.00%	0.00%	0.10%	4.60%	77.80%	6.80%	0.60%	0.60%	9.50%
	B	0.00%	0.00%	0.10%	0.20%	4.50%	74.60%	5.00%	3.30%	12.30%
	CCC/C	0.00%	0.00%	0.10%	0.20%	0.60%	12.50%	43.10%	28.30%	15.30%
Step 2. Global Corporate Average Transition Rates, adjusted for NR ratings **	**AAA**	89.90%	9.40%	0.50%	0.10%	0.10%	0.00%	0.10%	0.00%	
	AA	0.50%	90.80%	8.10%	0.50%	0.10%	0.10%	0.00%	0.00%	
	A	0.00%	1.70%	92.60%	5.20%	0.30%	0.10%	0.00%	0.10%	
	BBB	0.00%	0.10%	3.50%	91.90%	3.80%	0.50%	0.10%	0.20%	
	BB	0.00%	0.00%	0.10%	5.00%	86.00%	7.50%	0.60%	0.70%	
	B	0.00%	0.00%	0.10%	0.20%	5.20%	85.10%	5.70%	3.80%	
	CCC/C	0.00%	0.00%	0.10%	0.20%	0.60%	14.70%	50.90%	33.40%	
Step 3. Standard Normal bins	**AAA**	(∞, -1.27)	[-1.27, -2.41)	[-2.41, -2.81)	[-2.81, -2.88)	[-2.88, -3.15)	[-3.15, -3.28)	[-3.28, -5.61) *	[-1.27, -∞)	
	AA	(∞, 2.57)	[2.57, -1.36)	[-1.36, -2.49)	[-2.49, -2.96)	[-2.96, -3.08)	[-3.08, -3.34)	[-3.34, -3.53)	[2.57, -∞)	
	A	(∞, 3.42)	[3.42, 2.12)	[2.12, -1.58)	[-1.58, -2.6)	[-2.6, -2.9)	[-2.9, -3.18)	[-3.18, -3.28)	[3.42, -∞)	
	BBB	(∞, 5.61) *	[5.61, 3.1)	[3.1, 1.81)	[1.81, -1.69)	[-1.69, -2.44)	[-2.44, -2.77)	[-2.77, -2.93)	[5.61, -∞)	
	BB	(∞, 4.25)	[4.25, 3.4)	[3.4, 2.96)	[2.96, 1.63)	[1.63, -1.35)	[-1.35, -2.23)	[-2.23, -2.46)	[4.25, -∞)	
	B	(∞, 5.61) *	[5.61, 3.51)	[3.51, 3.08)	[3.08, 2.78)	[2.78, 1.6)	[1.6, -1.31)	[-1.31, -1.77)	[5.61, -∞)	
	CCC/C	(∞, 5.61) *	[5.61, 5.61) *	[5.61, 3.04)	[3.04, 2.73)	[2.73, 2.34)	[2.34, 1.01)	[1.01, -0.43)	[5.61, -∞)	

Notes:

*) For zero transition rates, a small manual adjustment (negative for cells below the main diagonal, and positive for cells above the main diagonal) is applied to obtain meaningful bin values.

**) The adjustment for not-rated assets (NR) is applied by proportionally increasing all other transition rates in each row.

[27] Bank for International Settlements. (2005). An explanatory note on the Basel II IRB risk weight functions. Retrieved 17 February 2023 from https://www.bis.org/bcbs/irbriskweight.pdf.

[28] Gupton, G., Finger, C. & Bhatia, M. (2007). CreditMetrics Technical Document. RiskMetrics Group. Section 8.5 Estimating asset correlations, pp 92-101. Retrieved 17 February 2023 from https://www.msci.com/documents/10199/93396227-d449-4229-9143-24a94dab122f.

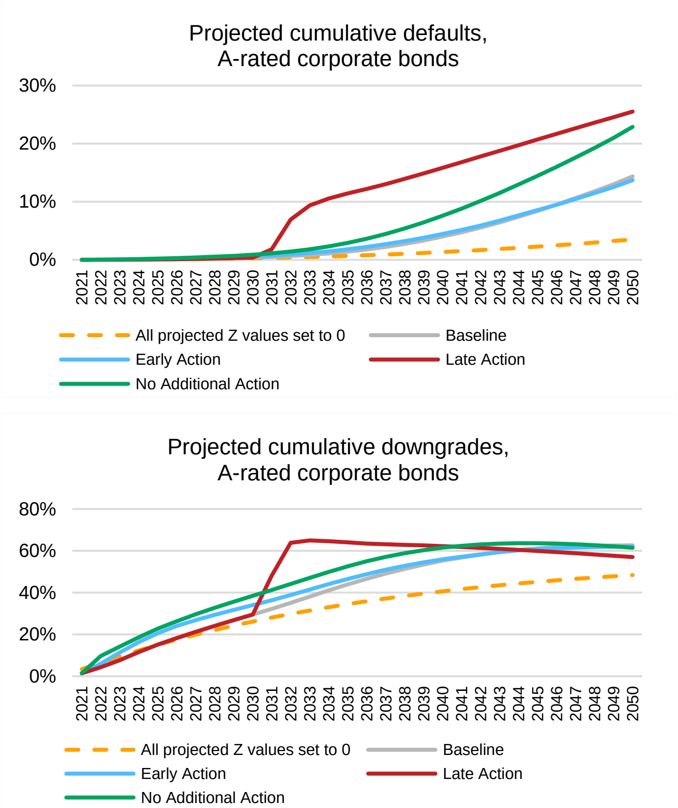
PROJECTED TRANSITION MATRICES

For each 2021 CBES scenario, we can project transition matrices based on projected systematic Z variables and asset correlation assumption. These are shown in **Appendix B**. Due to the large volume of outputs, projected transition matrices and associated Z values for selected durations only (2021, 2030, 2035 and 2050) are included.

One of the key outputs from our modelling are the cumulative[29] defaults and downgrades by credit rating. These can give insights about the impact of scenarios on the downgrade and default pathways for an asset portfolio. For example, they can help our understanding of the impact of a specific scenario to downgrades and defaults pathways; can help identify mitigating asset characteristics, such as credit rating, duration, industry, or currency; and can support the selection of appropriate management actions, such as portfolio selection and rebalancing.

For illustration, projected cumulative defaults and downgrades for A-rated corporate bonds are shown in **Figure 13** below, noting that for all other credit ratings projected pathways are similar.

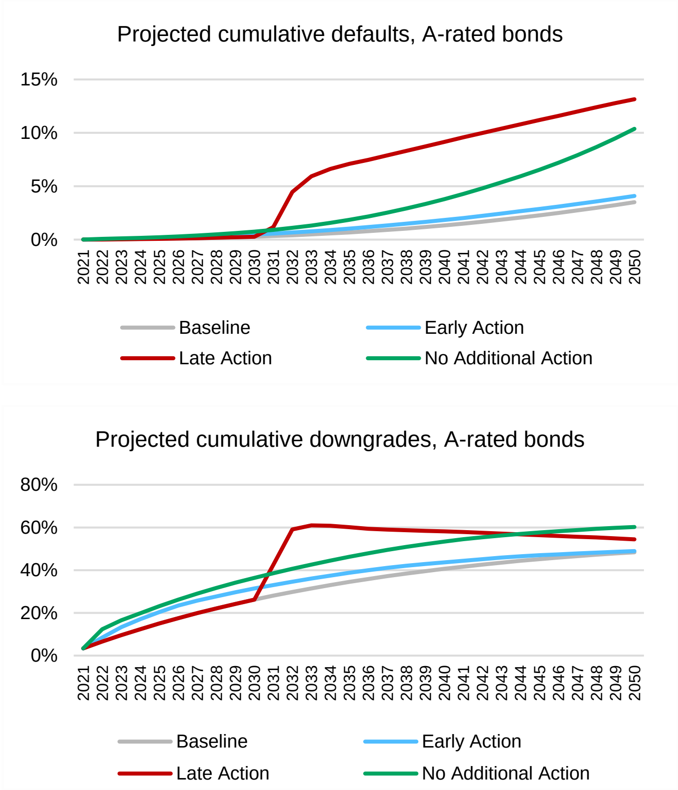
FIGURE 13. PROJECTED CUMULATIVE DEFAULTS AND DOWNGRADES, A-RATED CORPORATE BONDS, 2021 CBES SCENARIOS



The results are in line with projected Z variable pathways in **Figure 7**. In all scenarios, defaults and downgrades increase—except for downgrades in the Late Action scenario projection after 2032.

As mentioned earlier, the results can also be shown with projected Z value in the Baseline scenario set to zero and rescaled accordingly in all other scenarios. The resulting projected cumulative defaults and downgrades are lower, as shown in **Figure 14** below:

FIGURE 14. PROJECTED CUMULATIVE DEFAULT AND DOWNGRADE RATES, A-RATED CORPORATE BONDS, 2021 CBES SCENARIOS: Z VARIABLES IN THE BASELINE SCENARIO SET TO ZERO, RESCALED FOR ALL OTHER SCENARIOS



CREDIT RATING PATHWAYS

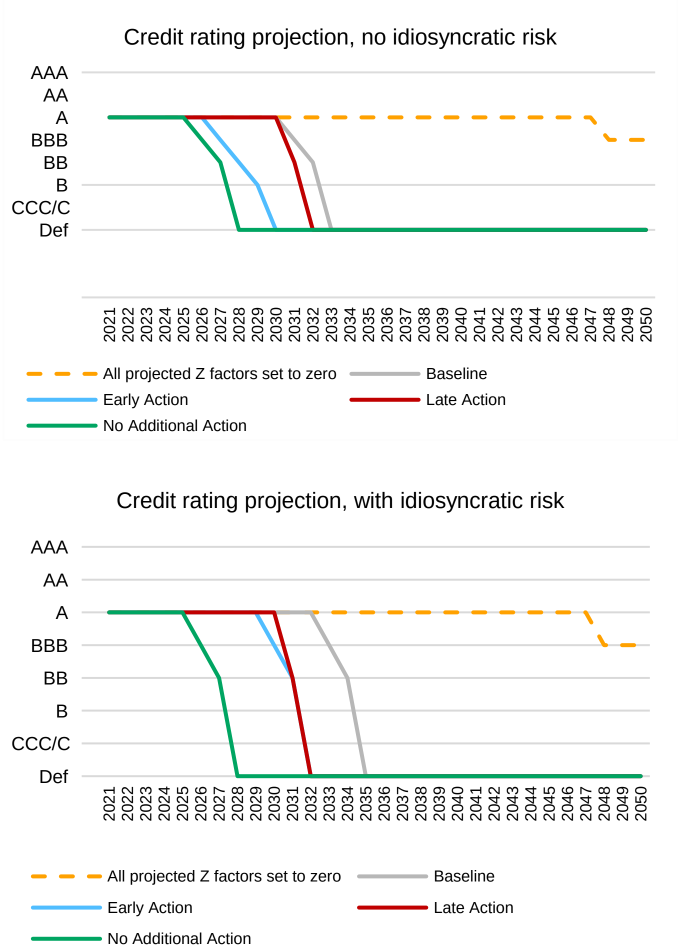
In this final modelling results section, we look at credit rating pathways. For illustration, we show results for corporate bonds rated A at the start of projection period (2021).

For completeness, we show results for corporate bonds only subject to systematic risk (e.g., corporate bonds with a risk profile like that of the overall global corporate bond portfolio), and corporate bonds subject to systematic and bond specific idiosyncratic risk (e.g., Crop and animal production corporate bonds).

[29] For the avoidance of doubt, 'cumulative' in the context of default and downgrade rates means that the underlying transition matrices are cumulative. 'Downgrades' are summed across all downgrades in a row – for example, the cumulative downgrade of A rated corporate bonds is the sum of downgrade rates from A to BBB, A to BB, A to B and A to CCC/C over the year.

The results are shown in **Figure 15** below:

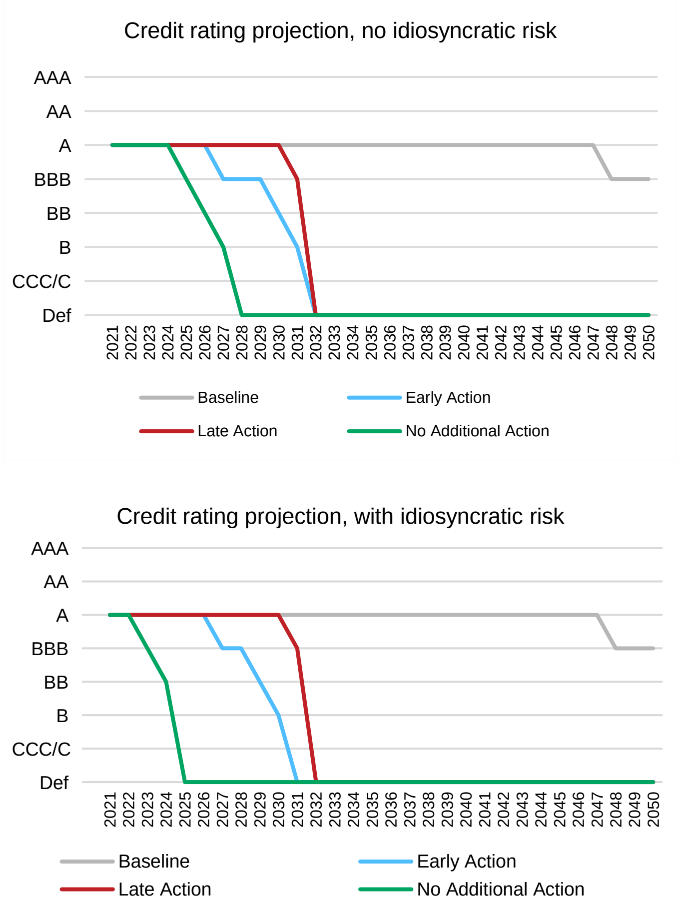
FIGURE 15. PROJECTED CREDIT RATINGS, A-RATED CORPORATE BONDS, 2021 CBES SCENARIOS



As expected, when only systematic risk is considered, credit rating pathways and projections of cumulative Z variables in **Figure 7** are broadly consistent. Adding idiosyncratic risk feeds through into slightly later default profiles for the Baseline and Early Action scenarios and has no apparent impact on the No Additional Action scenario. The elevated cumulative downgrades and defaults in the No Additional Action scenario in 2027 and 2028 means that the pathway to default remains unchanged.

We also show credit rating pathways with projected Z value in the Baseline scenario set to zero and rescaled in all other scenarios, in **Figure 16** below.

FIGURE 16. PROJECTED CREDIT RATINGS, A-RATED CORPORATE BONDS, 2021 CBES SCENARIOS: Z VARIABLES IN THE BASELINE SCENARIO SET TO ZERO, RESCALED FOR ALL OTHER SCENARIOS



When only systematic risk is considered, credit rating pathways remain broadly unchanged, and improve slightly in the Early Action scenario. Adding idiosyncratic risk feeds through a more accelerated pathway to default. The main reason is as shown in **Figure 15**: the Baseline scenario is further away to the right when adding idiosyncratic risk, i.e., idiosyncratic risk reduces corporate bond riskiness in the scenario. However, rescaling all variables relative to the Baseline scenario leads to a worsening of credit rating pathways in all other scenarios.

The analysis can be extended to show impact of other management actions, like rebalancing of defaulted corporate bonds. Illustrative results assuming defaulted corporate bonds are rebalanced back to the original credit rating in the year following default are shown in **Appendix D**.

Summary

Financial risks arising from climate change pose unique and significant challenges for portfolio management. In our paper, we applied a well-known structural credit risk model to 2021 CBES climate scenarios.

As part of our analysis, we considered several practical aspects related to corporate bond portfolios, such as portfolio exposures to global, systematic risks and asset-specific, idiosyncratic risks, and their relationship with the variables in the 2021 CBES scenarios. This gave useful and interesting insights into historical data and the projections of variables in the 2021 CBES.

Several conclusions can be drawn:

- Financial risks from climate change can have a severe, and possibly permanent, impact on the credit quality of corporate bonds.
- The earlier actions are taken to mitigate risks from climate change, the more likely it is that the implications for credit risk can be ameliorated over the long-term.
- The characteristics of financial risks from climate change (such as far-reaching impact in breadth and magnitude, an uncertain and long-term time horizon, and dependency on short-term action) require flexible, yet robust, approaches to model credit risks. The severe, volatile pathways of the variables in the 2021 CBES scenarios can result in new (even unforeseen) challenges to well-known, established credit risk models.
- The approach in our paper is, in places, intentionally simple, due mainly to time constraints. Yet we found the results to be very informative, in terms of possible future pathways of credit risk for a large corporate bond portfolio, and for an individual corporate bond.
- Where possible, analyses should consider practical considerations for asset portfolios. For example, rebalancing can have a material impact on results, as well as consideration for idiosyncratic risks.

Finally, we highlight several the actions that firms can take to address risks from the climate change:

- In order for results to be actionable and informative, ensure a clear understanding of financial risks from climate change and credit risk models.
- Ensure firms' approaches to modelling and projecting financial risks from climate change are holistic. This includes historical data, projection pathways, choices of (credit risk) models, parameters and assumptions.
- Use climate-related scenarios which are operationally tractable and can be incorporated in the business-as-usual models and processes.
- With the fast-growing body of academic literature and research on climate change and credit risk, ensure they keep up to date with any recent developments.

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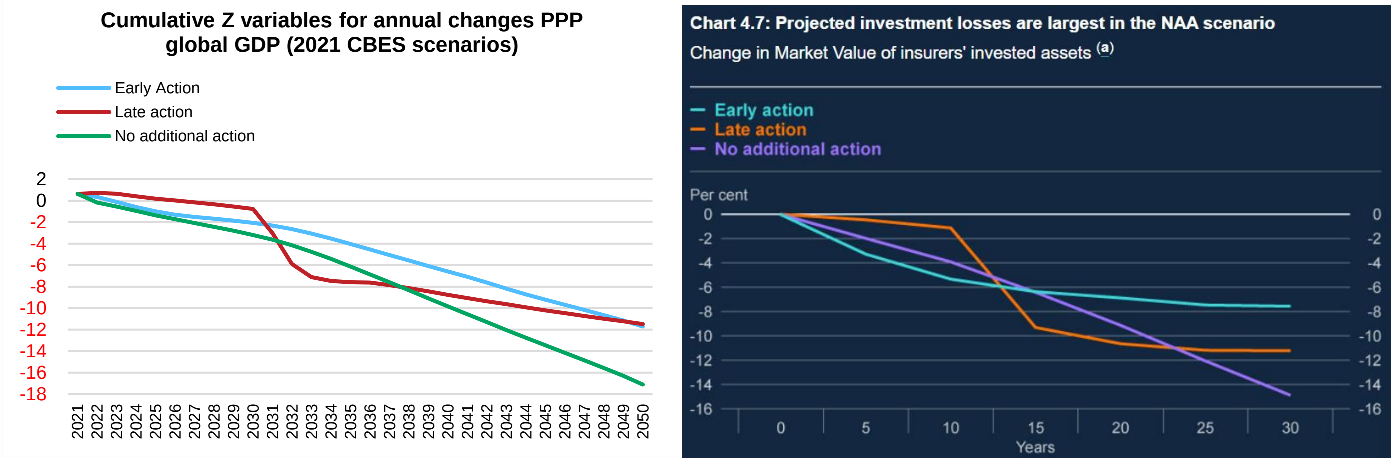
Appendix A. Comparison of cumulative Z variables with insurers' projected investment losses (2021 CBES)

In this appendix, we compare projected cumulative Z variables for annual changes in PPP global GDP variables (see **Figure 7**), shown on the left-hand side, with **Chart 4.7: Insurers' projected investment losses** in 2021 CBES results, as shown on the right-hand side below.

The results are similar in the Late Action and No Additional Action scenarios, and slightly different in the Early Action scenario.

FIGURE 17. COMPARISON OF CUMULATIVE Z VARIABLES, PPP GLOBAL GDP, WITH INSURERS' PROJECTED INVESTMENT LOSSES



Appendix B. Projected annual transition matrices at selected durations (2021, 2030, 2035 and 2050)

BASELINE

2021	Z=0.632	AAA	AA	A	BBB	BB	B	CCC/C	Def
	AAA	96.7%	3.2%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%
	AA	0.5%	96.9%	2.6%	0.1%	0.0%	0.0%	0.0%	0.0%
	A	0.0%	1.8%	96.7%	1.4%	0.0%	0.0%	0.0%	0.0%
	BBB	0.0%	0.1%	4.2%	94.7%	0.9%	0.1%	0.0%	0.0%
	BB	0.0%	0.0%	0.1%	6.4%	90.8%	2.5%	0.1%	0.1%
	B	0.0%	0.0%	0.1%	0.2%	6.6%	90.1%	2.2%	0.8%
	CCC/C	0.0%	0.0%	0.1%	0.2%	0.7%	20.3%	59.3%	19.5%

2030	Z = -0.218	AAA	AA	A	BBB	BB	B	CCC/C	Def
	AAA	91.1%	8.5%	0.3%	0.0%	0.0%	0.0%	0.0%	0.0%
	AA	0.1%	92.4%	7.2%	0.3%	0.0%	0.0%	0.0%	0.0%
	A	0.0%	0.5%	95.0%	4.3%	0.1%	0.0%	0.0%	0.0%
	BBB	0.0%	0.0%	1.3%	95.3%	3.0%	0.3%	0.0%	0.1%
	BB	0.0%	0.0%	0.0%	2.2%	90.2%	6.8%	0.4%	0.3%
	B	0.0%	0.0%	0.0%	0.0%	2.4%	89.4%	5.5%	2.7%
	CCC/C	0.0%	0.0%	0.0%	0.0%	0.2%	9.6%	54.5%	35.6%

2035	Z = -0.521	AAA	AA	A	BBB	BB	B	CCC/C	Def
	AAA	87.9%	11.5%	0.5%	0.0%	0.1%	0.0%	0.0%	0.0%
	AA	0.1%	89.7%	9.8%	0.4%	0.0%	0.0%	0.0%	0.0%
	A	0.0%	0.3%	93.3%	6.0%	0.2%	0.1%	0.0%	0.0%
	BBB	0.0%	0.0%	0.8%	94.2%	4.3%	0.4%	0.1%	0.1%
	BB	0.0%	0.0%	0.0%	1.5%	88.2%	9.2%	0.6%	0.6%
	B	0.0%	0.0%	0.0%	0.0%	1.6%	87.2%	7.2%	4.0%
	CCC/C	0.0%	0.0%	0.0%	0.0%	0.1%	7.0%	50.5%	42.3%

2050	Z = -0.656	AAA	AA	A	BBB	BB	B	CCC/C	Def
	AAA	86.2%	12.9%	0.6%	0.0%	0.1%	0.0%	0.0%	0.0%
	AA	0.0%	88.2%	11.1%	0.5%	0.0%	0.0%	0.0%	0.0%
	A	0.0%	0.2%	92.4%	7.0%	0.3%	0.1%	0.0%	0.0%
	BBB	0.0%	0.0%	0.7%	93.6%	5.0%	0.5%	0.1%	0.1%
	BB	0.0%	0.0%	0.0%	1.2%	87.0%	10.4%	0.7%	0.7%
	B	0.0%	0.0%	0.0%	0.0%	1.3%	85.9%	8.0%	4.8%
	CCC/C	0.0%	0.0%	0.0%	0.0%	0.1%	6.1%	48.5%	45.4%

EARLY ACTION

2021	Z=0.643	AAA	AA	A	BBB	BB	B	CCC/C	Def
	AAA	96.7%	3.2%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%
	AA	0.5%	96.9%	2.6%	0.1%	0.0%	0.0%	0.0%	0.0%
	A	0.0%	1.9%	96.7%	1.4%	0.0%	0.0%	0.0%	0.0%
	BBB	0.0%	0.1%	4.3%	94.7%	0.9%	0.1%	0.0%	0.0%
	BB	0.0%	0.0%	0.1%	6.5%	90.8%	2.5%	0.1%	0.1%
	B	0.0%	0.0%	0.1%	0.2%	6.7%	90.1%	2.2%	0.8%
	CCC/C	0.0%	0.0%	0.1%	0.2%	0.7%	20.4%	59.3%	19.3%

2030	Z = -0.198	AAA	AA	A	BBB	BB	B	CCC/C	Def
	AAA	91.2%	8.4%	0.3%	0.0%	0.0%	0.0%	0.0%	0.0%
	AA	0.1%	92.6%	7.0%	0.2%	0.0%	0.0%	0.0%	0.0%
	A	0.0%	0.5%	95.1%	4.2%	0.1%	0.0%	0.0%	0.0%
	BBB	0.0%	0.0%	1.4%	95.3%	2.9%	0.2%	0.0%	0.1%
	BB	0.0%	0.0%	0.0%	2.3%	90.3%	6.7%	0.4%	0.3%
	B	0.0%	0.0%	0.0%	0.0%	2.4%	89.5%	5.4%	2.7%
	CCC/C	0.0%	0.0%	0.0%	0.0%	0.2%	9.8%	54.7%	35.2%

2035	Z = -0.484	AAA	AA	A	BBB	BB	B	CCC/C	Def
	AAA	88.3%	11.1%	0.5%	0.0%	0.1%	0.0%	0.0%	0.0%
	AA	0.1%	90.1%	9.4%	0.4%	0.0%	0.0%	0.0%	0.0%
	A	0.0%	0.3%	93.6%	5.8%	0.2%	0.1%	0.0%	0.0%
	BBB	0.0%	0.0%	0.9%	94.4%	4.1%	0.4%	0.1%	0.1%
	BB	0.0%	0.0%	0.0%	1.5%	88.5%	8.9%	0.6%	0.5%
	B	0.0%	0.0%	0.0%	0.0%	1.6%	87.5%	7.0%	3.9%
	CCC/C	0.0%	0.0%	0.0%	0.0%	0.1%	7.3%	51.1%	41.5%

2050	Z = -0.553	AAA	AA	A	BBB	BB	B	CCC/C	Def
	AAA	87.5%	11.8%	0.5%	0.0%	0.1%	0.0%	0.0%	0.0%
	AA	0.1%	89.3%	10.1%	0.4%	0.0%	0.0%	0.0%	0.0%
	A	0.0%	0.3%	93.1%	6.2%	0.2%	0.1%	0.0%	0.0%
	BBB	0.0%	0.0%	0.8%	94.1%	4.5%	0.4%	0.1%	0.1%
	BB	0.0%	0.0%	0.0%	1.4%	87.9%	9.5%	0.6%	0.6%
	B	0.0%	0.0%	0.0%	0.0%	1.5%	86.9%	7.4%	4.2%
	CCC/C	0.0%	0.0%	0.0%	0.0%	0.1%	6.8%	50.0%	43.0%

LATE ACTION

2021 Z=0.632	AAA	AA	A	BBB	BB	B	CCC/C	Def
AAA	96.7%	3.2%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%
AA	0.5%	96.9%	2.6%	0.1%	0.0%	0.0%	0.0%	0.0%
A	0.0%	1.8%	96.7%	1.4%	0.0%	0.0%	0.0%	0.0%
BBB	0.0%	0.1%	4.2%	94.7%	0.9%	0.1%	0.0%	0.0%
BB	0.0%	0.0%	0.1%	6.4%	90.8%	2.5%	0.1%	0.1%
B	0.0%	0.0%	0.1%	0.2%	6.6%	90.1%	2.2%	0.8%
CCC/C	0.0%	0.0%	0.1%	0.2%	0.7%	20.3%	59.3%	19.5%

2030 Z = -0.218	AAA	AA	A	BBB	BB	B	CCC/C	Def
AAA	91.1%	8.5%	0.3%	0.0%	0.0%	0.0%	0.0%	0.0%
AA	0.1%	92.4%	7.2%	0.3%	0.0%	0.0%	0.0%	0.0%
A	0.0%	0.5%	95.0%	4.3%	0.1%	0.0%	0.0%	0.0%
BBB	0.0%	0.0%	1.3%	95.3%	3.0%	0.3%	0.0%	0.1%
BB	0.0%	0.0%	0.0%	2.2%	90.2%	6.8%	0.4%	0.3%
B	0.0%	0.0%	0.0%	0.0%	2.4%	89.4%	5.5%	2.7%
CCC/C	0.0%	0.0%	0.0%	0.0%	0.2%	9.6%	54.5%	35.6%

2035 Z = -0.108	AAA	AA	A	BBB	BB	B	CCC/C	Def
AAA	92.0%	7.6%	0.3%	0.0%	0.0%	0.0%	0.0%	0.0%
AA	0.1%	93.3%	6.4%	0.2%	0.0%	0.0%	0.0%	0.0%
A	0.0%	0.6%	95.5%	3.7%	0.1%	0.0%	0.0%	0.0%
BBB	0.0%	0.0%	1.6%	95.5%	2.6%	0.2%	0.0%	0.0%
BB	0.0%	0.0%	0.0%	2.6%	90.7%	6.1%	0.3%	0.3%
B	0.0%	0.0%	0.0%	0.0%	2.7%	89.9%	5.0%	2.4%
CCC/C	0.0%	0.0%	0.0%	0.0%	0.2%	10.7%	55.7%	33.3%

2050 Z = -0.245	AAA	AA	A	BBB	BB	B	CCC/C	Def
AAA	90.8%	8.8%	0.3%	0.0%	0.0%	0.0%	0.0%	0.0%
AA	0.1%	92.2%	7.4%	0.3%	0.0%	0.0%	0.0%	0.0%
A	0.0%	0.5%	94.9%	4.4%	0.1%	0.0%	0.0%	0.0%
BBB	0.0%	0.0%	1.3%	95.2%	3.1%	0.3%	0.1%	0.1%
BB	0.0%	0.0%	0.0%	2.1%	90.1%	7.0%	0.4%	0.3%
B	0.0%	0.0%	0.0%	0.0%	2.3%	89.2%	5.6%	2.8%
CCC/C	0.0%	0.0%	0.0%	0.0%	0.2%	9.4%	54.2%	36.2%

NO ADDITIONAL ACTION

2021 Z=0.643	AAA	AA	A	BBB	BB	B	CCC/C	Def
AAA	96.7%	3.2%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%
AA	0.5%	96.9%	2.6%	0.1%	0.0%	0.0%	0.0%	0.0%
A	0.0%	1.9%	96.7%	1.4%	0.0%	0.0%	0.0%	0.0%
BBB	0.0%	0.1%	4.3%	94.7%	0.9%	0.1%	0.0%	0.0%
BB	0.0%	0.0%	0.1%	6.5%	90.8%	2.5%	0.1%	0.1%
B	0.0%	0.0%	0.1%	0.2%	6.7%	90.1%	2.2%	0.8%
CCC/C	0.0%	0.0%	0.1%	0.2%	0.7%	20.4%	59.3%	19.3%

2030 Z = -0.378	AAA	AA	A	BBB	BB	B	CCC/C	Def
AAA	89.5%	10.0%	0.4%	0.0%	0.1%	0.0%	0.0%	0.0%
AA	0.1%	91.1%	8.5%	0.3%	0.0%	0.0%	0.0%	0.0%
A	0.0%	0.4%	94.2%	5.1%	0.2%	0.1%	0.0%	0.0%
BBB	0.0%	0.0%	1.1%	94.8%	3.6%	0.3%	0.1%	0.1%
BB	0.0%	0.0%	0.0%	1.8%	89.2%	8.0%	0.5%	0.4%
B	0.0%	0.0%	0.0%	0.0%	1.9%	88.3%	6.4%	3.4%
CCC/C	0.0%	0.0%	0.0%	0.0%	0.1%	8.2%	52.5%	39.1%

2035 Z = -0.694	AAA	AA	A	BBB	BB	B	CCC/C	Def
AAA	85.8%	13.4%	0.6%	0.1%	0.1%	0.0%	0.0%	0.0%
AA	0.0%	87.8%	11.5%	0.5%	0.0%	0.1%	0.0%	0.0%
A	0.0%	0.2%	92.1%	7.2%	0.3%	0.1%	0.0%	0.0%
BBB	0.0%	0.0%	0.6%	93.4%	5.2%	0.5%	0.1%	0.1%
BB	0.0%	0.0%	0.0%	1.1%	86.6%	10.8%	0.8%	0.7%
B	0.0%	0.0%	0.0%	0.0%	1.2%	85.5%	8.3%	5.0%
CCC/C	0.0%	0.0%	0.0%	0.0%	0.1%	5.8%	47.8%	46.3%

2050 Z = -0.801	AAA	AA	A	BBB	BB	B	CCC/C	Def
AAA	84.3%	14.7%	0.7%	0.1%	0.1%	0.0%	0.0%	0.0%
AA	0.0%	86.5%	12.7%	0.6%	0.1%	0.1%	0.0%	0.0%
A	0.0%	0.2%	91.2%	8.1%	0.3%	0.1%	0.0%	0.0%
BBB	0.0%	0.0%	0.5%	92.7%	5.8%	0.6%	0.1%	0.2%
BB	0.0%	0.0%	0.0%	1.0%	85.4%	11.8%	0.9%	0.9%
B	0.0%	0.0%	0.0%	0.0%	1.0%	84.3%	9.0%	5.6%
CCC/C	0.0%	0.0%	0.0%	0.0%	0.1%	5.1%	46.1%	48.7%

Appendix C. Proof of the lower bound of the common correlation of n random variables

Consider the variance of the sum of n unit variance random variables X_i, i =1,…, n. We have that:

$$var\left(\sum_{i=1}^{n} X_i\right) = \sum_{i=1}^{n} var(X_i) + \sum_{i=1}^{n}\sum_{j\neq i}^{n} cov(X_i, X_j)$$

Since $var(X_i) = 1$ and $cov(X_i, X_j) = \rho_{X_i,X_j} = \rho$, we have that:

$$var\left(\sum_{i=1}^{n} X_i\right) = n + \sum_{i=1}^{n}\sum_{j\neq i}^{n} \rho_{X_i,X_j} = n + n(n-1)\rho$$

Finally, since $var(\sum_{i=1}^{n} X_i) \geq 0$, we readily get that:

$$0 \leq n + n(n-1)\rho$$

which is equivalent to:

$$\rho \geq -\frac{1}{n-1} \text{ Q.E.D.}$$

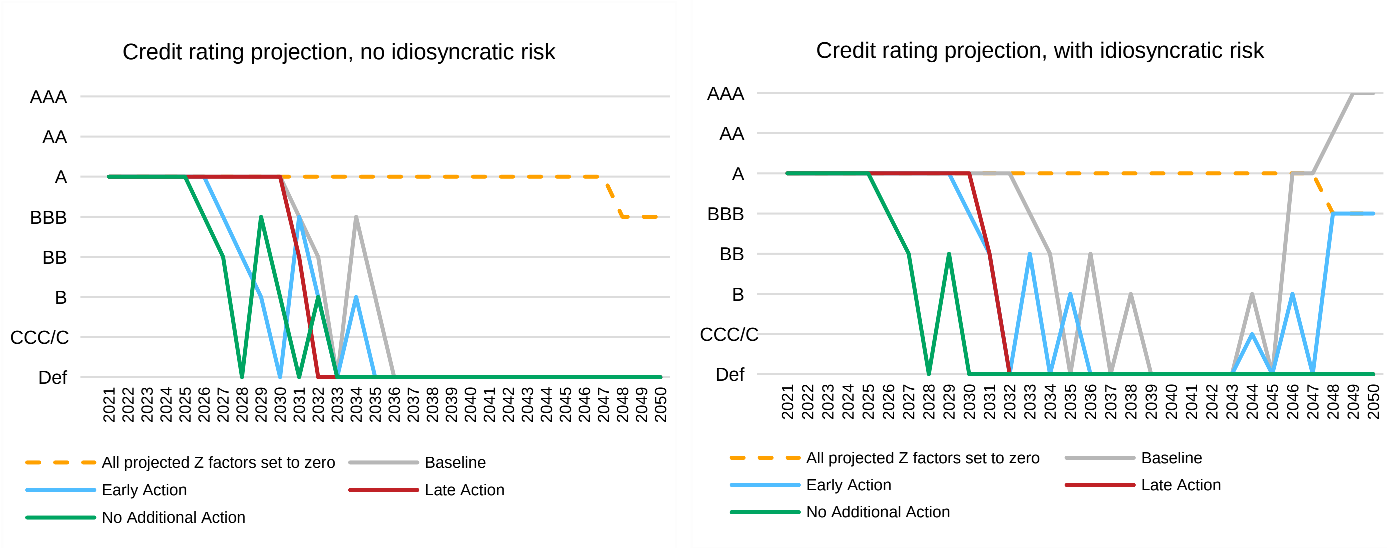
Appendix D. Credit rating pathways with rebalancing

Rebalancing of corporate bond portfolios is an important aspect of investment management, and approaches can vary considerably.

To illustrate the impact of rebalancing, we repeat the analysis in **Figure 15. Projected credit ratings, A-rated corporate bonds, 2021 CBES scenarios**, but assuming that defaulted corporate bonds are replaced with A-rated corporate at the beginning of the year following default.

We assume no rebalancing or trading costs, and no other cost or tax items are allowed for. The results are shown below.

FIGURE 18. PROJECTED CREDIT RATINGS FOR A-RATED CORPORATE BONDS, WITH REBALANCING, 2021 CBES SCENARIOS



As expected, credit rating pathways are more volatile when adding rebalancing of the portfolio. As the portfolio credit riskiness is large, when we consider systematic risk only, by year 2036 A-rated corporate bonds default in all scenarios.

The addition of idiosyncratic risk reduces portfolio credit riskiness, and eventually credit ratings bounce back in the Baseline and Early Action scenarios.